PERTH MODERN SCHOOL Exceptional schooling. Exceptional students. Independent Public School	Year 12 Specialist TEST 2 2018 TIME: 5 mins reading 40 minutes working Classpads **allowed!** 36 marks 8 Questions

Name: SOLUTIONS

Teacher: ______________________

Note: All part questions worth more than 2 marks require working to obtain full marks.

Q1 (2 & 2 = 4 marks)

Consider $f(x) = x^3 - x^2 + 4x - 4$.

i) Show that $(x-2i)$ is a factor of $f(x)$

$f(2i) = (2i)^3 - (2i)^2 + 4(2i) - 4 \Rightarrow -8i + 4 + 8i - 4 = 0$

$= 0$

✓ identifies $f(2i)$ ✓ equates to zero

ii) Determine three linear factors of $f(x)$

$(x-1)(x-2i)(x+2i)$

✓ $x-1$

✓ $x+2i$

Q2 (5 marks)

Consider $f(x) = x^3 + bx^2 + cx + 8$ where b & c are constants. Given that $(x+2)$ is a factor of $f(x)$ and when $f(x)$ is divided by $(x-3)$ has a remainder of -10. Determine b & c.

$f(-2) = 0$ $f(3) = -10$

$-8 + 4b - 2c + 8 = 0$ $-10 = 27 + 9b + 3c + 8$

$4b - 2c = 0$

$9b + 3c = -45$

$b = -3$

$c = -6$

✓ $f(-2) = 0$

✓ $f(3) = -10$

✓ shows two simultaneous equ

✓ state b

✓ state c

f/f

Q3 (3 marks)

Given that $f(x) = \sqrt{x+2}$ and $g(x) = 5x - 3$. Does $f \circ g(x)$ exist over the natural domain of g? Explain your answer.

$d_g: R$ $r_g: R$

$d_f: x \geq -2$ $r_f: y \geq 0$

To exist $r_g \subseteq d_f$

$R \not\subseteq x \geq -2$

$\therefore$ does not exist over natural domain

✓ States conclusion

✓ States domain of f

✓ States range of g

Q4 (2 & 2 = 4 marks)

Given that $f(x)=\sqrt{x}$ and $h(x)=\dfrac{1}{x^2+5}$:

i) Determine the rule of $h \circ f(x)$ $= \dfrac{1}{x+5}$

✓ states rule
✓ states simplified rule

ii) State the natural domain and range of $h \circ f(x)$

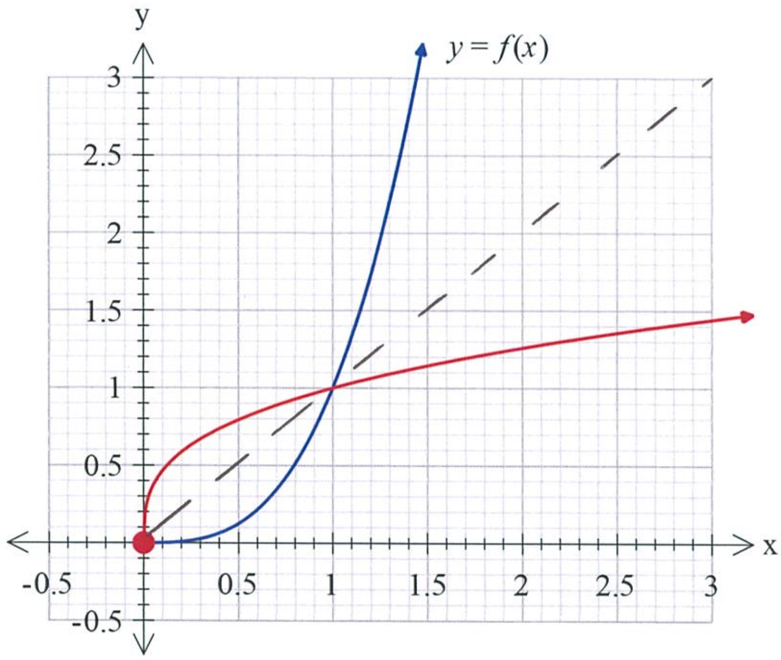
$x \geq 0$ $y \leq \frac{1}{5}$

✓ states domain
✓ states range

f/t

Q5 (3 & 3 = 6 marks)

i) On the diagram, sketch the inverse function $f^{-1}(x)$

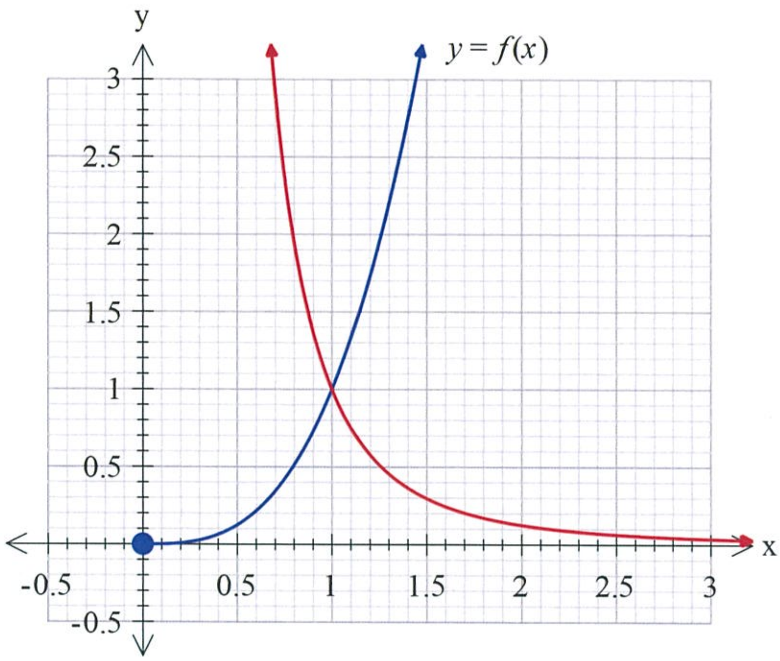


✓ function reflected in line $y=x$

✓ intersects $f(x)$ at $(1,1)$

✓ $y < 1.5$ over $0 < x < 3$

ii) On the diagram below, sketch $y = \dfrac{1}{f(x)}$



✓ asymptote at $x=0$
✓ asymptote at $y=0$
} No need to give rule

✓ intersects $f(x)$ at $(1,1)$

Q6) (1, 1, 2 & 2= 6 marks)

Consider the function $f(x) = \dfrac{cx+d}{ax+b}$ where $a, b, c \,\&\, d$ are non-zero constants.

i) Determine the natural domain of f

$x \neq -\dfrac{b}{a}$ ✓

ii) Determine the limit that f approaches as $x \to \pm\infty$

$y \to \dfrac{c}{a}$ ✓

iii) Determine the inverse function $f^{-1}(x)$ in terms of $a, b, c \,\&\, d$.

$x = \dfrac{cy+d}{ay+b}$

$x(ay+b) = cy+d$

$axy + xb = cy + d$

$y(ax-c) = d - xb$

$f^{-1}(x) = \dfrac{-xb+d}{ax-c}$

✓ interchanges x + y

✓ determines rule.

iv) Determine the possible values of $a, b, c \,\&\, d$ if $f = f^{-1}$.

$b = -c$

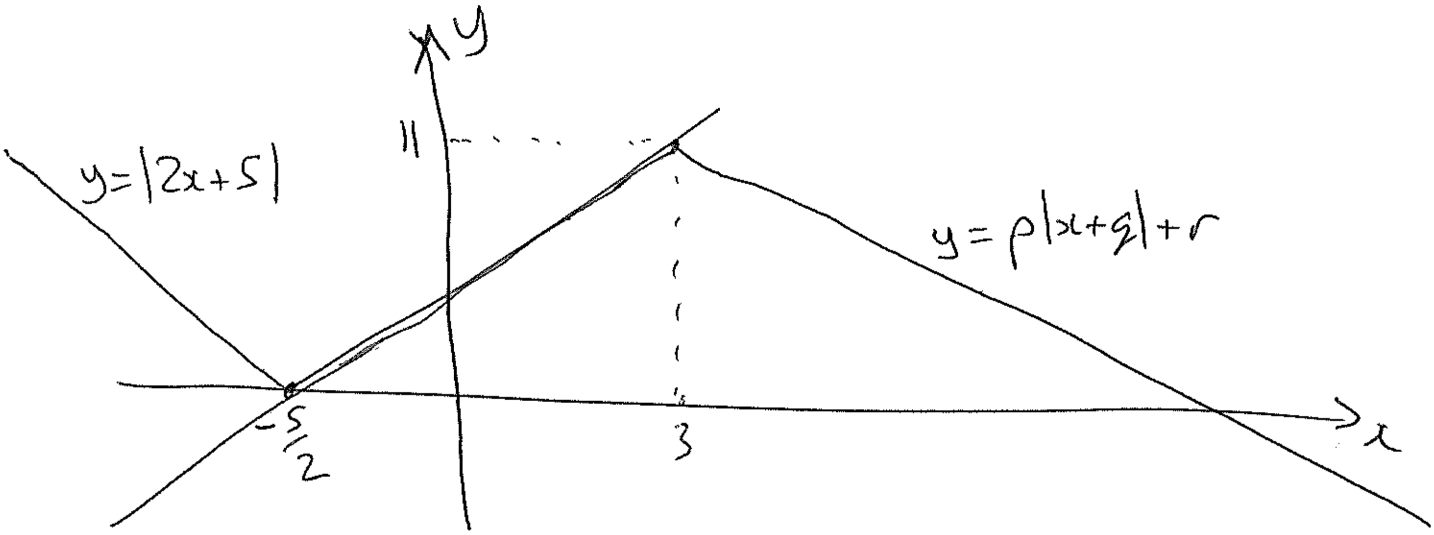
✓ states $b = -c$

✓ no restrictions on a + d.

Q7 (4 marks)

Consider the equation $|2x+5| = p|x+q|+r$ which is true and only true for $\frac{-5}{2} \le x \le 3$.

Determine the possible values of the constants $p, q \,\&\, r$.



$q = -3$

$r = 11$

$(-\frac{5}{2}, 0)$

$0 = p\left|-\frac{5}{2} - 3\right| + 11$

$-11 = \frac{11}{2}p$

$p = -2$

✓ sketches overlap only between $-\frac{5}{2} \le x \le 3$ (OR other reasoning that shows this)

$$\left.\begin{array}{l} ✓\ q = -3 \\ ✓\ r = 11 \\ ✓\ p = -2 \end{array}\right\}$$

Right/wrong

No follow through

Q8 (4 marks)

Let $z = \cos(2\theta) + i\sin(2\theta)$, prove that $\dfrac{1+z}{1-z} = \dfrac{i}{\tan\theta}$

$$LHS = \frac{\cos 2\theta + 1 + i\sin 2\theta}{1 - \cos 2\theta - i\sin 2\theta} \cdot \frac{(1-\cos 2\theta) + i\sin 2\theta}{(1-\cos 2\theta) + i\sin 2\theta}$$

$$= \frac{(1+\cos 2\theta)(1-\cos 2\theta) - \sin^2 2\theta + i\sin 2\theta\,(2)}{(1-\cos 2\theta)^2 + \sin^2 2\theta}$$

$$= \frac{1 - \cos^2\theta - \sin^2 2\theta + 2i\sin 2\theta}{1 - 2\cos 2\theta + \cos^2 2\theta + \sin^2 2\theta}$$

$$= \frac{2i\sin 2\theta}{2 - 2\cos 2\theta}$$

$$= \frac{i\sin 2\theta}{2\sin^2\theta} \qquad (\cos 2\theta = 1 - 2\sin^2\theta)$$

$$= \frac{2i\sin\theta\cos\theta}{2\sin\theta\sin\theta}$$

$$= \frac{i\cos\theta}{\sin\theta}$$

$$= \frac{i}{\tan\theta}$$

✓ multiplies by conjugate of denominator

✓ shows that resulting numerator is complex

✓ obtains expression in terms of θ by using double angle formulae

✓ obtains $\dfrac{i\cos\theta}{\sin\theta}$